Fully polarized states and decoherence

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Abstract

The aim of this review is to show how "ferromagnetic" states, that is, states having a fully polarization, can produce intrinsic decoherence by unitary evolution. This effect can give an understanding of recent experiments on mesoscopic devices as quantum point contacts showing the 0.7 conductance anomaly and the wide number of data about saturation of dephasing time observed at very low temperatures, as a fully polarized two dimensional electron gas. But similar effects can be seen in different area of physics as for example the Dicke model describing the interaction of two-level systems with a radiation mode. In this case one can show that decoherence is intrinsic and remove a Schrödinger cat state leaving a single coherent state, collapsing the wave function in the thermodynamic limit. So, saturation of dephasing time at low temperatures in mesoscopic devices can be understood by a fully polarized two dimensional electron gas that, by an exchange model, can be reduced to a generalized form of the Dicke Hamiltonian and where the quasiparticles are spin excitations interacting with magnons. In this way, one can see that several experiments on nanowires and quantum dots can be satisfactorily explained. The existence of intrinsic decoherence in the thermodynamic limit could have deep implications in fundamental problems like quantum measurement and irreversibility. Recent experiments with cavities with a large number of photons and with nuclear magnetic resonance in organic molecular crystals give a first strong support to this view.

1 Introduction

Decoherence, intended as the disappearance of interference terms and the emerging of the classical world by the unavoidable interaction of a quantum systems with the degrees of freedom of an external environment, is gaining increasing acceptance as a paradigm to understand how classicality emerges from a quantum world [1]. Notwithstanding the increasing acceptance of this ideas, several criticisms have been put forward [2]. Particularly, the common belief that decoherence could resolve the measurement problem in quantum mechanics cannot be supported. Besides, the environment is quantum itself and all the times we work out realistic examples, as we will see below, quantum electrodynamics with a large number of degrees of freedom is implied. So, one may ask if the emergence of classicality and the lost of quantum coherence is rather an intrinsic effect of unitary evolution in the proper limits.

Recent experiments on mesoscopic devices [3, 4, 5, 6] seem to indicate that a new mechanism is at work producing decoherence at very small temperature. At present, it is still an open question if the effect is an intrinsic one [7, 8, 9]. Besides, an anomaly in the conductance in quantum point contacts, the so called 0.7 anomaly, has prompted several people to put forward a hypothesis of a polarized electron gas [10, 11, 12, 13]. The same mechanism may be at work in the former case [14, 15]. A further decoherence effect in quantum dots, due to hyperfine interaction between electrons and nuclei, has been predicted [16, 17, 18] assuming a fully polarized state for nuclei.

The existence of polarized states in mesoscopic devices has recently been supported by an experiment on a two-dimensional electron gas (2DEG) [19]. Analysis based on numerical computations were done about [20] showing the possible existence of such a state in 2DEG. Theoretical studies also supported the existence of a ferromagnetic state in nanowires [21, 22, 23], that could be favored by the low density.

The relevance of polarized states to determine the very behavior at zero temperature of mesoscopic devices could indeed reveal itself fundamental to understand a lot of issues about the foundations of quantum mechanics. The question to be asked is if decoherence should be considered as an effect of quantum evolution and if the "collapse" of the wave function could be considered a new physical effect. A recent proposal for an experiment to clarify this situation has been recently put forward [24]. The aim is to see whether some stochastic

effect is at work producing the “collapse”.

Recently, we pointed out how quantum electrodynamics is able to remove quantum coherence in the thermodynamic limit [25, 26, 27, 28, 29, 30, 31]. Two relevant assumptions are at foundation of this conclusion. Firstly, we take a fully polarized state of two-level atoms. Secondly, we take the thermodynamic limit. Both these assumptions are essential for the proper working of an intrinsic mechanism of decoherence. If such a mechanism does exist one can draw conclusions on the problem of quantum measurement and irreversibility.

Quantum measurement problem is a basic one in quantum mechanics and appeared since the start with the Copenaghen interpretation. The essential point relies on the boundary between a quantum and a classical word. That this boundary could be taken to infinity has been shown recently with some experiments realized by Haroche's group [32, 33]. These experiments confirm an understanding of quantum measurement in the thermodynamic limit, taking the number of photons increasing, puts forward by Gea-Banacloche [34, 35] and further analyzed by Knight and Phoenix [36, 37]. These experiments and theoretical analysis seem to give a first hint toward the relevance of the thermodynamic limit in a real understanding of the measurement problem in quantum mechanics.

The existence of an arrow of time is a basic problem in physics as also the Schrödinger equation is reversible in time. This appears an old problem since the controversy between Boltzmann and Loschmidt. Our conclusions rely on the assumption that a many-body effect in the thermodynamic limit is acting removing reversibility. Essentially, our argument is based on the very existence of singular limits in time, that is, periodic functions having fastly oscillating phases in time are averaged away being blurred. Singular limits have been pointed out by Berry [38] for the transition from a quantum to a classical world and pioneered, for the problem of measurement in quantum mechanics, by Bohm [39].

A strong experimental evidence for this instability, proper to many-body quantum systems, has been shown in NMR experiments with organic molecular crystals by Pastawski, Usaj and Levenstein[40, 41, 42, 43, 44]. These authors were able to prove that the decoherence rate dependends just on the coupling constant between spins and their number. The decoherence appears as an intrinsic effect as expected by our approach with the proper dependencies and is rapidly attained in the thermodynamic limit, while the decay deviates from ordinary

exponential to gaussian.

So, polarized states could be essential for the proper understanding of fundamental issues in physics, relying the solution on quantum mechanical interactions between spins or between two-level systems and bosonic modes.

The paper is structured in this way. In sec.2 we give an introduction to decoherence by unitary evolution in the thermodynamic limit. In sec.3 we introduce the Dicke model and we analyze it both for a larger number of bosons and fermions. In sec.4 we consider two exchange models for a conduction electron into a polarized background and the Holstein-Primakoff transformation. In sec.5 we discuss the implications of dynamical decoherence for the problem of irreversibility and the theory of quantum measurement. In sec.6 the conclusions are given.

2 Decoherence in the thermodynamic limit

There is a deep relation between quantum mechanics and statistical mechanics. A first and well-known connection is between the time evolution operator e^{-iHt} ($\hbar = 1$) and the density matrix $e^{-\beta H}$, being H the Hamiltonian of the system. The simple change $t \leftrightarrow -i\beta$ gives the connection. So, one may ask how far such a relation could be taken. In statistical mechanics, a privileged role is played by the thermodynamic limit to recover the laws of thermodynamics [45]. This limit is generally taken by having the number of particles and volume going to infinity and keeping the density constant. Anyhow, for some systems, the limit of the number of particles going to infinity is enough.

One may think to extend the concept of thermodynamic limit to quantum mechanics assuming that, instead of the laws of thermodynamics, this limit recovers classical mechanics. Indeed, one can prove the following theorem, for non-interacting quantum systems as happens for a perfect gas in thermodynamic [27]:

Theorem 1 (Classicality) *An ensemble of N non interacting quantum systems, for properly chosen initial states, has a set of operators $\{A_i, B_i, \ldots : i = 1 \ldots N\}$ from which one can derive a set of observables behaving classically.*

A deep conceptual similarity is given through this theorem between statistical mechanics and quantum mechanics for non-interacting systems. The proof is given by showing that, for a proper chosen initial state, the quantum fluctuations are negligible with respect to the average values of the observables that evolves in time in the thermodynamic limit. Two points are essential to the proof: The thermodynamic limit and the proper chosen initial state.

Decoherence is obtained, after the theorem above, once the following statement is proved:

Theorem 2 (Decoherence) *An ensemble of N non interacting quantum systems, having a set of operators $\{A_i, B_i, \ldots : i = 1 \ldots N\}$, from which one can derive a set of observables behaving classically, interacting with a quantum system through a Hamiltonian having a form like $V_0 \otimes \sum_{i=1}^N A_i$, can produce decoherence if properly initially prepared in a fully "polarized" state.*

To prove this theorem we consider a Hamiltonian like

$$H = H_S + \sum_{i=1}^N H_i + V_0 \otimes \sum_{i=1}^N A_i \tag{1}$$

being H_S the Hamiltonian of the quantum system, H_i the Hamiltonian of the i-th system in the bath and V_0 the interaction operator with the observables A_i of the bath. Then, passing to the interaction picture for the bath one has

$$H_I = H_S + NV_0 \otimes \sum_{i=1}^N \hat{A}(t) \tag{2}$$

having introduced the operator

$$\hat{A}(t) = \frac{1}{N}\sum_{j=1}^N e^{iH_jt} A_j e^{-iH_jt} \tag{3}$$

and we assume to exist the limit $N \to \infty$ for this operator at least on the chosen initial state of the bath $|\chi\rangle$. This state is a fully polarized state as we take all the systems in the bath in the same eigenstate of the Hamiltonian H_i. These two hypothesis are essential for our proof.

Then, if the thermodynamic limit $N \to \infty$ is formally taken on the Hamiltonian H_I, we enter a strongly coupling regime [46, 47, 48, 49] having at the leading order the operator

$$H_I' = NV_0 \otimes \hat{A}(t) \tag{4}$$

that can be diagonalized by the following unitary transformation

$$U_0(t) = \sum_n e^{-iNv_n\bar{a}t}|v_n\rangle\langle v_n||\chi\rangle\langle\chi| \tag{5}$$

being $V_0|v_n\rangle = v_n|v_n\rangle$ having assumed a discrete spectrum for the sake of simplicity and $\bar{a} = \langle\chi|\hat{A}(t)|\chi\rangle = constant$. Then, the density matrix of the quantum system in the bath becomes

$$\rho_S(t) = \sum_n |\langle v_n|\psi_S(0)\rangle|^2|v_n\rangle\langle v_n| + \sum_{n\neq m}\langle v_m|\psi_S(0)\rangle\langle\psi_S(0)|v_n\rangle|v_n\rangle\langle v_m|e^{-iN(v_n-v_m)\bar{a}t}. \tag{6}$$

being $|\psi_S(0)\rangle$ the initial state of the quantum system. To complete the proof, in the thermodynamic limit, a meaning should be attached to the oscillating terms in the interference contribution in the density matrix. Here we face an interesting problem. In order to recover an oscillating term we should be able to proper sample it. Otherwise, if the sampling happens with a too small frequency, the numerical rounding on the phases changes it into noise due to the large time step. This matter can be easily derived from the sampling theorem [50]. This means that as N increases to infinity, to recover the oscillating terms becomes increasingly difficult as we are going to probe even more smaller time scales and the numerical precision needed in time step to recover the phases is even more demanding. So, in the limit, we get just a noisy term with null average added to the mixed part of the density matrix and the interfering part of the density matrix gives no contribution in the thermodynamic limit, completing the proof of the theorem.

The conclusion to be drawn from the above about fastly oscillating functions is that *if a function has even more smaller time scales, e.g. due to the large number of particles, phases are scrambled due to the need of a large sampling frequency, and a noisy behavior with a null average is obtained.*

Two further consideration are needed here. Firstly, we have provided another way to produce a random sequence of numbers (see fig.1). Secondly, one may ask, from a physical standpoint, if a meaning can be attached to unitary evolution when this is probing Planck time scales. E.g., loop quantum gravity [51] does not support unitary evolution at Planck time scale.

The impossibility to recover completely phases by a quantum system with a large number of particles grants irreversibility as an intrin-

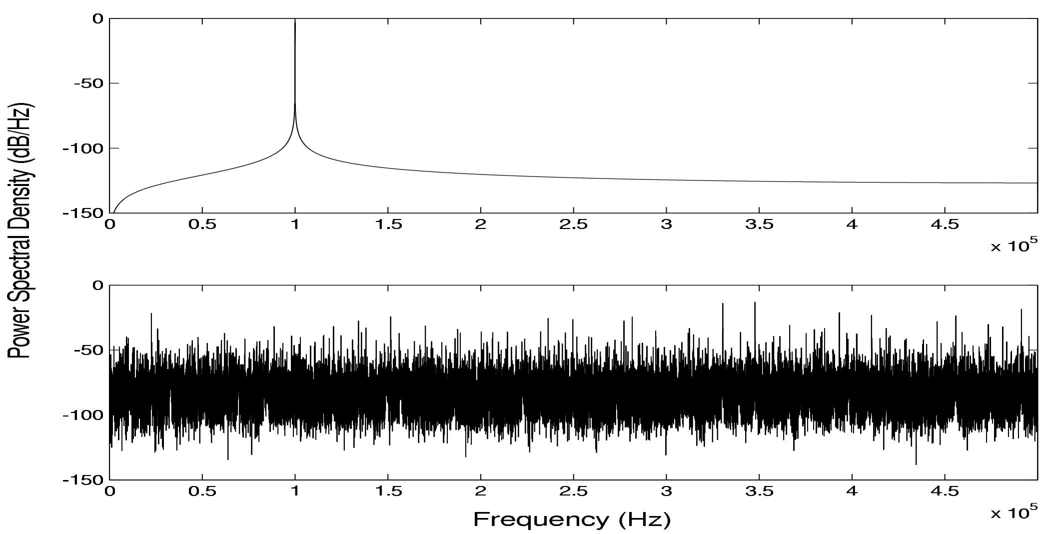

Figure 1: Periodograms of the function $\sin(2\pi N t)$ with $N = 10^5$ Hz (upper) and $N = 10^{43}$ Hz (lower) both sampled with a frequency of 1 MHz.

sic effect. The above theorems grant that also decoherence is intrinsic. So, one appears the cause of the other.

3 Decoherence in a bosonic-fermionic model

One of the fundamental models in quantum electrodynamics, describing the interaction between a single radiation mode with an ensemble of two-level atoms, is the Dicke model [52] described by the Hamiltonian

$$H = \Delta S_z + \omega a^\dagger a + 2g S_x (a + a^\dagger) \tag{7}$$

being Δ the separation between the two levels of the atoms, $a^\dagger$ and a the creation and annihilation operators for the radiation mode with frequency ω, g the coupling constant that depends on the volume where the radiation stays as $V^{-\frac{1}{2}}$ and

$$S_x = \frac{1}{2}\sum_{n=1}^{N} \sigma_{xi} \tag{8}$$

$$S_z = \frac{1}{2}\sum_{n=1}^{N}\sigma_{zi}$$

with σ_{xi} and σ_{zi} the Pauli matrices for the i-th atom.

We analyzed this model in the strong coupling regime [25, 28]. On this basis we showed that the leading order should be given by

$$H_0 = \omega a^\dagger a + 2gS_x(a + a^\dagger) \tag{9}$$

that can be immediately diagonalized giving the time evolution operator

$$U_0(t) = e^{i\frac{4S_x^2 g^2}{\omega^2}(\omega t - \sin(\omega t))} e^{2S_x[\beta(t)a^\dagger - \beta^*(t)a]} \tag{10}$$

being $\beta(t) = \frac{g}{\omega}(1 - e^{i\omega t})$ and treating S_x as a c-number. It is immediately realized that, for a fully polarized state $|\chi\rangle$ such that $S_x|\chi\rangle = -\frac{N}{2}|\chi\rangle$ and the radiation mode in the ground state $|0\rangle$ one has, in the thermodynamic limit $N \to \infty$ and constant volume, a coherent state for a classical radiation field

$$|\psi\rangle = e^{i\frac{N^2 g^2}{\omega^2}(\omega t - \sin(\omega t))}| - N\beta(t)\rangle|\chi\rangle. \tag{11}$$

A polarized state for the atomic ensemble has produced the effect to amplify the quantum fluctuations of the ground state of the radiation mode to the classical level, an effect we called quantum amplifier (QAMP). The state of the ensemble of two-level atoms is left untouched and acts passively. Higher order corrections where also given [26, 28, 31].

Results about the classicality in the thermodynamic limit of observables as the number operator and "spin" in the Dicke model were given in pioneering papers by Hepp and Lieb [53, 54]. So, classicality seems a rather general property of this model than the strong coupling approximation seems to grant. Recently, an in-depth study by Brandes and Emary [55, 56], proved that the Dicke model undergoes a quantum phase transition. These authors considered the limit with the number of particles and the volume going to infinity and keeping the density constant. In this case, one can introduce the coupling parameter as $g = \frac{\lambda}{\sqrt{N}}$ and the thermodynamic limit just amounts to take the limit $N \to \infty$, remembering that g depends on the volume as $V^{-\frac{1}{2}}$ with the Hamiltonian being

$$H = \Delta S_z + \omega a^\dagger a + 2\frac{\lambda}{\sqrt{N}}S_x(a + a^\dagger). \tag{12}$$

The quantum phase transition happens in the parameter space at $\lambda_c = \frac{\sqrt{\Delta\omega}}{2}$. To reach their aim, Brandes and Emary used the Holstein-Primakoff transformation [57] given by

$$
\begin{aligned}
S_+ &= \sqrt{N} b^\dagger \left(1 - \frac{b^\dagger b}{N}\right)^{\frac{1}{2}} \qquad (13)\\
S_- &= \sqrt{N} \left(1 - \frac{b^\dagger b}{N}\right)^{\frac{1}{2}} b\\
S_z &= b^\dagger b - \frac{N}{2}
\end{aligned}
$$

that, assuming a maximal spin state, gives a series in $\frac{1}{N}$ with the Hamiltonians at various orders

$$
\begin{aligned}
H_0 &= \Delta b^\dagger b + \omega a^\dagger a + \lambda(a^\dagger + a)(b^\dagger + b) \qquad (14)\\
H_1 &= -\frac{\lambda}{2N}(b^\dagger b^\dagger b + b^\dagger b b)\\
&\dots .
\end{aligned}
$$

It easy to see that the leading order Hamiltonian, describing the behavior of the model in the thermodynamic limit, can be easily diagonalized. So, in this limit, the Dicke model is exactly integrable. These authors reached two relevant conclusions that are fundamental for our aims. Firstly, they showed that any macroscopic coherence disappears in the thermodynamic limit. Secondly, by a numerical effort, they proved that, keeping N fixed and letting the coupling constant increase as $\lambda \to \infty$, the model is well described by the Hamiltonian

$$
H_{lim} = \omega a^\dagger a + 2\frac{\lambda}{\sqrt{N}} S_x (a^\dagger + a) \qquad (15)
$$

with the proper quantization axis being given by S_x. This latter result gives a strong support to our above work. But this conclusions can be pushed further and to prove analitically that [31], also at constant volume, *in the limit of $N \to \infty$ or $g \to \infty$ or both, the Hamiltonian H_{lim} becomes exact and the Dicke model becomes integrable again.* This can be proved straightforwardly by applying the Holstein-Primakoff transformation, this time, with respect to S_x axis as

$$
S_x = -\frac{N}{2} + c^\dagger c \qquad (16)
$$

$$
\begin{aligned}
S_+ &= \sqrt{N}c^\dagger\left(1-\frac{c^\dagger c}{N}\right)^{\frac{1}{2}} \\
S_- &= \sqrt{N}\left(1-\frac{c^\dagger c}{N}\right)^{\frac{1}{2}}c.
\end{aligned}
$$

being $S_z = \frac{1}{2}(S_+ + S_-)$. We note that this transformation cannot be applied in the problem studied by Brandes and Emary while their approach cannot be applied here. These methods just complete each other with the boundary being given by the Hamiltonian H_{lim}. One has in this way, turning back to the coupling constant g,

$$
H = \omega a^\dagger a - Ng(a+a^\dagger) + 2gc^\dagger c(a+a^\dagger) + \frac{\Delta}{2}\sqrt{N}(c^\dagger+c) - \frac{\Delta}{4}\frac{1}{\sqrt{N}}(c^\dagger c^\dagger c + c^\dagger cc) + \ldots. \tag{17}
$$

that is not easy to manage unless the unitary transformation

$$
U_0 = e^{\frac{2g}{\omega}c^\dagger c(a-a^\dagger)} \tag{18}
$$

is applied giving the Hamiltonian at the various orders

$$
\begin{aligned}
H_0' &= \omega a^\dagger a - Ng(a+a^\dagger) + 4N\frac{g^2}{\omega}c^\dagger c \qquad (19) \\
H_1' &= \frac{\Delta}{2}\sqrt{N}\left[c^\dagger e^{-\frac{2g}{\omega}(a-a^\dagger)} + ce^{\frac{2g}{\omega}(a-a^\dagger)}\right] \\
H_2' &= -4\frac{g^2}{\omega}(c^\dagger c)^2 \\
H_3' &= -\frac{\Delta}{4N}\left[c^\dagger c^\dagger c e^{-\frac{2g}{\omega}(a-a^\dagger)} + c^\dagger cce^{\frac{2g}{\omega}(a-a^\dagger)}\right] \\
&\vdots
\end{aligned}
$$

proving our initial assertion for $N \to \infty$. The prove for $g \to \infty$ can be obtained directly by the Rayleigh-Schrödinger series [31].

The main question to ask now is if the Hamiltonian H_{lim} can indeed provide decoherence. The point here is delicate and involves again the concept of a singular limit. As a matter of fact, we already showed that can generate classical state of radiation field producing a new effect we named QAMP. But let us consider a phase Schröndiger cat state for the radiation field defined as[58]

$$
|\zeta\rangle = \mathcal{N}(|\gamma e^{i\phi}\rangle + |\gamma e^{-i\phi}\rangle) \tag{20}
$$

being γ and ϕ two real parameters, $\mathcal{N}$ a normalization constant. These are two superposed coherent states of the radiation field. So, we can apply the unitary evolution (10) to this state obtaining

$$|\psi(t)\rangle = e^{i\xi(t)}\mathcal{N}(e^{i\phi_1(t)}|\frac{Ng}{\omega}(e^{i\omega t}-1)+\gamma e^{i\phi-i\omega t}\rangle+e^{i\phi_2(t)}|\frac{Ng}{\omega}(e^{i\omega t}-1)+\gamma e^{-i\phi-i\omega t}\rangle)|0\rangle_- \tag{21}$$

where a ground state has been assumed for the other bosonic mode built from the two-level atoms. This result appears rather surprising as, taking formally the limit $N \to \infty$, the superposition is removed leaving just a classical state of radiation. This could claimed only if the quantum effect due to the initial state would be really removed and not just displaced to infinity or hidden by the level of description. Indeed, infinity is not a really comfortable place to do physics and this can be seen by considering the interfering part of the Wigner function of the above state, given by

$$\begin{aligned} W_{INT} &= \frac{2}{\pi}\exp\left[-\left(x+\frac{\sqrt{2}Ng}{\omega}(1-\cos(\omega t))-\sqrt{2}\gamma\cos(\phi)\cos(\omega t)\right)^2\right] \\ &\times \exp\left[-\left(p+\frac{\sqrt{2}Ng}{\omega}\sin(\omega t)+\sqrt{2}\gamma\cos(\phi)\sin(\omega t)\right)^2\right] \\ &\times \cos\left[2\sqrt{2}\gamma\sin(\phi)\left(p\sin(\omega t)-x\cos(\omega t)\right)+\gamma^2\sin(2\phi)+8\gamma\frac{Ng}{\omega}\sin(\phi)(1-\cos(\omega t))\right]. \end{aligned} \tag{22}$$

and we easily recognize a singular limit as happened into the prove of the theorem on decoherence in the thermodynamic limit. Again we can get noise with null average in the thermodynamic limit and we have true decoherence.

The relevance of the above analysis relies on the fact that this study implies a realistic model of the radiation-matter interaction that has wide applicability. This takes us to the important conclusions that electromagnetic interaction is the main vehicle of decoherence and, being also the means used to do a measurement, we get an understanding of why quantum coherence gets lost in a measurement device. It is essential to point out again that our conclusions rely on the two fundamental hypothesis of a fully polarized state and the thermodynamic limit.

4 Decoherence in an exchange model

The discovery of weak localization [59] permitted to realize some experiments to measure the coherence time in mesoscopic devices. First experiments pointed out that, differently from what the theory predicts, a saturation of the dephasing time was seen [60, 61, 62]. Initially, this was not considered a concern as some external effects could easily explain the saturation. In 1997 an experiment by Mohanty, Jariwala and Webb, realized in order to eliminate any possible external effects, proved that the saturation of the dephasing time seems to be intrinsic. This started a debate that is still open [7, 8, 9] with the fundamental question to be answered if the effect is really intrinsic or not. This is a very fundamental question as hits the possibility to realize fully coherent mesoscopic devices.

Curiously enough, the debate considers just some devices neglecting e.g. the situation on quantum dots where one has two contrasting experiments [5, 6] but both confirming the saturation. The discrepancy between the two experiments relies on the fact that in [5] was observed a dependence of saturation time on the number of electrons in the two dimensional electron gas of the dot, while in [6] this dependence was not seen. But it should be said that different approaches were used to interpret the data. If a dependence on the number of electrons exists then this also means a dependence on the size, being the density kept constant.

A dependence on the size for the saturation of dephasing time in nanowires has been also experimentally observed [4]. While this geometrical dependence should be confirmed by further experiments, no experimental proposal seemed to address this point so far. But some experimental and numerical results in mesoscopic physics for 2DEG, nanowires, nanoclusters and so on [19, 20, 21, 22, 23, 64] point out toward a possibility that the Fermi electron gas in this devices may be polarized. Besides, a similar hypothesis was suggested for the 0.7 conductance anomaly [10, 11, 12, 13] even if an objection has been put forward here based on a theorem by Lieb and Mattis[63]. A polarized Fermi gas can easily explain the dependence on geometry of the saturation time.

Let us see this in a simple model. We consider as a possible Hamiltonian [14]

$$H = \frac{\Delta}{2}\sigma_z - J\sigma_x \sum_{i=1}^{N} \tau_{xi} \tag{23}$$

where we consider negligible the dynamical part of the N spins that interact with a single one through the coupling constant J. Δ is the separation between the energy levels of the single spin. Both σ and τ represent Pauli matrices. The sign of the coupling constant J does not matter in this case. This appears as a simplified exchange model describing the interaction between a conduction electron and the Fermi gas. It is straightforward to write down the unitary evolution as

$$U_0(t) = \exp\left[-it\left(\frac{\Delta}{2}\sigma_z - J\sigma_x \sum_{i=1}^{N} \tau_{xi}\right)\right] \tag{24}$$

that can be disentangled as [65]

$$U_0(t) = \exp\left(-i\hat{\Lambda}(t)\sigma_+\right)\exp\left(-\ln\hat{\Sigma}(t)\sigma_3\right)\exp\left(-i\hat{\Lambda}(t)\sigma_-\right) \tag{25}$$

being the operators

$$\hat{\Lambda}(t) = \frac{-J\sum_{i=1}^{N}\tau_{xi}}{\hat{\Omega}}\frac{\sin(\hat{\Omega}t)}{\hat{\Sigma}(t)} \tag{26}$$

$$\hat{\Sigma}(t) = \cos(\hat{\Omega}t) + i\frac{\Delta}{2\hat{\Omega}}\sin(\hat{\Omega}t) \tag{27}$$

$$\hat{\Omega}(t) = \sqrt{\frac{\Delta^2}{4} + J^2\left(\sum_{i=1}^{N}\tau_{xi}\right)^2}. \tag{28}$$

Assuming a fully polarized state, in the lower spin state, for the Fermi gas $|\chi\rangle$ and N being enough large, one has the following wave function for the conduction electron

$$|\psi(t)\rangle =\approx \exp\left(-itNJ\sigma_x\right)|\psi(0)\rangle. \tag{29}$$

So, it is easy to see that, when the conduction electron is in the lower eigenstates of σ_z one gets a spin coherent state [65]. But we are interested in the density matrix of the conducting electron. This can be written as

$$\rho_{1,1}(t) = \frac{1-\cos(2NJt)}{2} \tag{30}$$

$$\rho_{1,-1}(t) = -i\frac{1}{2}\sin 2NJt \tag{31}$$

$$\rho_{-1,1}(t) = i\frac{1}{2}\sin 2NJt \tag{32}$$

$$\rho_{-1,-1}(t) = \frac{1+\cos(2NJt)}{2}. \tag{33}$$

We can easily recognize that we can have oscillation with increasing frequency in the limit $N \to \infty$. We have again a singular limit and the time scale for it is given by

$$T_s = \frac{\pi}{NJ}. \tag{34}$$

It should be noted at this point that, if the diffusion constant can be taken as $D \sim J$, we recover the experimental results by Lin and Kao [66] but here the dependence on the geometry was not tested. Finally, we have showed as a simple exchange model with the condition of a singular limit recovers the dependence on the number of electrons N or, keeping the density constant, on the geometry. The averaged density matrix permits us to recover classical probabilities reducing the density matrix to a mixed form, i.e. we have decoherence.

We can make the above model more realistic and consider an exchange model like [67]

$$H = H_0 + H_h + H_e \tag{35}$$

being

$$H_0 = \sum_{\mathbf{p}\sigma} E_{\mathbf{p}} c^\dagger_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} \tag{36}$$

the Hamiltonian describing the itinerant electrons,

$$H_h = -J_h \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j \tag{37}$$

is the Heisenberg term of ferromagnetic type, $J_h > 0$, representing the interaction between the spins of the gas, and

$$H_e = J \sum_i \mathbf{S}_i \cdot \mathbf{s}_i \tag{38}$$

is the exchange term (a Kondo term as from the first Hund's rule), being

$$\mathbf{s}_i = \sum_{\alpha\beta} c^\dagger_{i\alpha} \mathbf{s}_{\alpha\beta} c_{i\beta} \tag{39}$$

with $\mathbf{s}_{\alpha\beta}$ spin matrices whose components for spin $\frac{1}{2}$ are given by $\frac{\boldsymbol{\sigma}_{\alpha\beta}}{2}$ with $\boldsymbol{\sigma}_{\alpha\beta}$ the Pauli matrices. The sign of the coupling constant J in the exchange term should be determined on a physical ground. Besides, in order to favor the tendency of the conduction electron to align, we neglect H_0.

In order to study this model in a situation of a fully polarized electron gas, we use again the Holstein-Primakoff transformation as

$$
\begin{aligned}
S_i^+ &= a_i^\dagger \left(2S - a_i^\dagger a_i\right)^{\frac{1}{2}} \\
S_i^- &= \left(2S - a_i^\dagger a_i\right)^{\frac{1}{2}} a_i \\
S_i^+ &= S - a_i^\dagger a_i .
\end{aligned}
\tag{40}
$$

We arrive at the following expression, omitting H_0 as assumed initially,

$$
H' = -2zNJ_hS^2 + \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} a_{\mathbf{k}}^\dagger a_{\mathbf{k}} + JS\sum_i s_i^z + J\sqrt{\frac{S}{2}}\sum_{\mathbf{k}}\left(a_{\mathbf{k}}^\dagger s_{\mathbf{k}}^- + a_{\mathbf{k}} s_{\mathbf{k}}^+\right)
\tag{41}
$$

being $\epsilon_{\mathbf{k}} = 2zJ_hS(1-\gamma_{\mathbf{k}})$ and $\gamma_{\mathbf{k}} = \frac{1}{z}\sum_{\mathbf{a}} e^{i\mathbf{k}\cdot\mathbf{a}}$ with $\mathbf{a}$ the vector linking two nearest neighbor spins and z the number of nearest neighbor spins. We recognize here the Dicke model seen in the preceding section, with the main difference that we have a large number of modes and the spin operator depending on $\mathbf{k}$. This means that the known considerations about radiation-matter interaction apply and we can have Rabi oscillations for a small number of modes becoming a decaying exponential, i.e. an effect of decoherence, when the number of modes becomes larger. Indeed, putting

$$
s_{\mathbf{k}}^+ = \frac{1}{\sqrt{N}}\sum_i s_i^+ e^{i\mathbf{k}\cdot\mathbf{r}_i}
\tag{42}
$$

and similarly for $s_{\mathbf{k}}^-$, instead of itinerant electrons, we have quasi-particles being spin excitations, described by the Hamiltonian

$$
H_S = JS\sum_i s_i^z = \frac{JS}{2}\sum_{\mathbf{k}}\left(c_{\mathbf{k}\uparrow}^\dagger c_{\mathbf{k}\uparrow} - c_{\mathbf{k}\downarrow}^\dagger c_{\mathbf{k}\downarrow}\right),
\tag{43}
$$

interacting with magnons. Finally, passing to interaction picture we get

$$
H_I = J\sqrt{\frac{S}{2}}\sum_{\mathbf{k}}\left(a_{\mathbf{k}}^\dagger s_{\mathbf{k}}^- e^{i(\epsilon_{\mathbf{k}}-JS)t} + a_{\mathbf{k}} s_{\mathbf{k}}^+ e^{-i(\epsilon_{\mathbf{k}}-JS)t}\right)
\tag{44}
$$

and we can immediately identify to the leading order the processes that can induce decoherence, that is, we can have an itinerant electron to flip its spin by emitting a magnon or, being a magnon present, by absorption. We can conclude that the only possible choice for

the coupling is $J > 0$ proper to a Kondo Hamiltonian. So, we can immediately apply the Fermi golden rule to the above Hamiltonian obtaining the rate of spin flip

$$\Gamma = 2\pi \frac{J^2 S}{2} \sum_{\mathbf{k}} \delta(\epsilon_{\mathbf{k}} - JS). \tag{45}$$

Passing from the sum to the integral, a crucial step, we get, for a two dimensional device,

$$\Gamma_{d=2} = \frac{1}{2} V m^* J^2 S \tag{46}$$

where the dependence on the geometry is evident in the volume V and we have assumed

$$\epsilon_{\mathbf{k}} = \frac{\mathbf{k}^2}{2m^*} \tag{47}$$

for the dispersion relation, being m^* the effective mass of the electron. We note that a proper experimental verification of this theory relies on the assumption that the diffusion coefficient D is kept constant during measurement as the quantity measured is the coherence length L_ϕ related to the dephasing time τ_ϕ by the relation $L_\phi = \sqrt{D\tau_\phi}$. At the present, the only experiment that does this is the one by Natelson's group [4] that gives the right behavior on geometry. Experiments on quantum dots give directly the dephasing time but, as said above, contrasting results were obtained. Then, further experimental work is needed.

5 Quantum measurement and irreversibility

The Dicke model represents a truly representative description of radiation-matter interaction in the non-relativistic limit. This model permits to prove that decoherence is intrinsic to unitary evolution when two more conditions are taken: a proper initial state and the thermodynamic limit. Besides, we have seen that recovering phases can be very demanding when the thermodynamic limit is taken as the time scale becomes even more smaller. Sampling such rapidly oscillating functions is not physically realizable and we have to consider averages in time. This has the main effect to remove indeterminacy on Schrödinger cat states as we have proved. Besides, superposition states are reduced to a single pure state. This points to consider the radiation-matter

interaction as the main vehicle for decoherence. Indeed, it is really difficult to see a measurement device that does not use electromagnetic interactions for its aims.

This implies that there exists an intrinsic instability in the determination of the phases in quantum mechanics that is properly acting when the thermodynamic limit is taken. This effect seems to have been seen in recent NMR experiment made by Pastawski, Levenstein and Usaj[40, 41, 42, 43, 44]. The aim of this authors was rather different with respect to ours. Indeed, the existence of the irreversibility, a rather common experience in the macroscopic world, is a longstanding problem in physics that can be traced back to the controversy between Loschmidt and Boltzmann (see [40] and refs. therein). The essential point is that all the fundamental laws of physics appear to be reversible in time, so whatever process one can realize in one direction of time, is perfectly legal when time is reversed. Then, it is very difficult to see how, from such a symmetry, the existence of entropy or of the Boltzmann H function can be obtained. One could think to reverse all the velocities of the particles and turn back to the initial state, reversing the natural order-disorder sequence that is never observed.

In order to solve this problem, Boltzmann introduced the statistical hypothesis of "molecular chaos" (*Stosszahlansatz*) [68], assuming that the reversing claimed by Loschmidt is not physically realizable. We know today that chaos inherent to Hamiltonian systems as expressed by the Kolmogorov-Arnold-Moser theorem [69], can give a proper answer at the level of classical mechanics to the Boltzmann's hypothesis. But we also know that world is not classical but quantum and we have to find a similar mechanism in quantum mechanics to support Boltzmann's hypothesis and explain irreversibility. The problem we meet here is that chaos does not exist in quantum mechanics, being the Schrödiger equation linear.

Experiments by Pastawski, Levenstein and Usaj have shown that, in NMR with organic molecular crystals as ferrocene, cobaltocene and others, reversibility for this spin systems is not recovered and the proper dephasing time, being the system unable to recover the initial phase, decreases as the thermodynamic limit is taken, proving that the effect is even more important in this limit. This is exactly the same behavior we have derived from both the exchange models in the preceding section. Another interesting result in their experiments is that the decay is gaussian, differently from the exponential decay

expected by ordinary decoherence effects.

An interesting point here is how physically realized is the thermodynamic limit. The explanation is given by the existence of phase transitions in condensed matter. Statistical mechanics requires, from a mathematical standpoint, the thermodynamic limit for the existence of phase transitions [70, 71]. For all practical purposes, as our everyday experience proves, Avogadro number is enough to have a thermodynamic limit.

Dephasing, as also shown in the Dicke model where it provokes decoherence in the thermodynamic limit, is then at the root of the quantum measurement problem and irreversibility as initially conceived by Boltzmann. Scrambling of rapidly oscillating phases due to the rough sampling is the cause of both the effects, being effectively noise.

6 Conclusions

The conclusions to be drawn are that both on experimental and theoretical sides there are strong hints for the existence of a many-body effect that would permits to understand the origin of irreversibility and implies decoherence in the unitary evolution.

The main assumptions we have made to recover such an effect is the thermodynamic limit and a fully polarized state, that is, we need a proper initial preparation of the quantum system.

So, further confirmations of the relevance of the thermodynamic limit in quantum mechanics would extend significantly the theory, removing a somewhat undefined concept of environment, as conceived today. So, quantum mechanics should be seen as the bootstrap program of our classical world.

References

[1] W. H. Zurek, Rev. Mod. Phys. **75**, 715 (2003) and refs. therein.

[2] M. Schlosshauer, quant-ph/0312059 and refs. therein.

[3] P. Mohanty, E. M. Q. Jariwala and R. A. Webb, Phys. Rev. Lett. **78**, 3366 (1997).

[4] D. Natelson, R. L. Willett, K. W. West and L. N. Pfeiffer, Phys. Rev. Lett. **86**, 1821 (2001).

[5] D. P. Pivin, Jr., A. Andresen, J. P. Bird and D. K. Ferry, Phys. Rev. Lett. **82**, 4687 (1999).

[6] A. G. Huibers, J. A. Folk, S. R. Patel, C. M. Marcus, C. I. Duruõz and J. S. Harris, Jr., Phys. Rev. Lett. **83**, 5090 (1999).

[7] F. Pierre, A. B. Gougam, A. Anthore, H. Pothier, D. Esteve and N. O. Birge, Phys. Rev. B **68**, 085413 (2003).

[8] P. Mohanty and R. A. Webb, Phys. Rev. Lett. **91**, 066604 (2003).

[9] F. Pierre, and N. O. Birge, cond-mat/0401182.

[10] C. K. Wang and K. F. Berggren, Phys. Rev. B 57, 4552 (1998).

[11] K. Hirose and N. S. Wingreen, Phys. Rev. B 64, 073305 (2001).

[12] K. Hirose, S. S. Li, and N. S.Wingreen, Phys. Rev. B 63, 033315 (2001).

[13] H. Bruus, V. V. Cheianov, and K. Flensberg, Physica E **10**, 97 (2001).

[14] M. Frasca, Physica E **15**, 252 (2002).

[15] M. Frasca, Phys. Rev. B **68**, 193413 (2003).

[16] A. Khaetskii, D. Loss, and L. Glazman, Phys. Rev. Lett. **88**, 186802 (2002).

[17] J. Schliemann, A. Khaetskii, and D. Loss, Phys. Rev. B **66**, 245303 (2002).

[18] J. Schliemann, A. Khaetskii, and D. Loss, J. Phys.: Condens. Matter **15**, R1809 (2003).

[19] A. Ghosh, C. J. B. Ford, M. Pepper, H. E. Beere, and D. A. Ritchie, cond-mat/0310628, to appear on Phys. Rev. Lett.

[20] C. Attaccalite, S. Moroni, P. Gori-Giorgi, and G. B. Bachelet, Phys. Rev. Lett. **88**, 256601 (2002); (Erratum) Phys. Rev. Lett. **91**, 109902 (2003).

[21] A. Delin and E. Tosatti, Phys. Rev. B **68**, 144434 (2003).

[22] A. Delin, E. Tosatti, and R. Weht, Phys. Rev. Lett. **92**, 057201 (2004).

[23] A. Delin and E. Tosatti, cond-mat/0401524.

[24] W. Marshall, C. Simon, R. Penrose, and D. Bouwmeester, Phys. Rev. Lett. **91**, 130401 (2003); (Erratum) Phys. Rev. Lett. **91**, 159903 (2003).

[25] M. Frasca, Phys. Lett. A **283**, 271 (2001); (Erratum) Phys. Lett. A **306**, 184 (2002).

[26] M. Frasca, J. Opt. B: Quantum Semiclass. Opt. **4**, S443 (2002).

[27] M. Frasca, Phys. Lett. A **308**, 135 (2003).

[28] M. Frasca, J. Phys. B: At. Mol. Opt. Phys. **36**, 2757 (2003).

[29] M. Frasca, Ann. Phys. **306**, 193 (2003).

[30] M. Frasca, J. Phys. B: At. Mol. Opt. Phys. **37**, 1273 (2004).

[31] M. Frasca, quant-ph/0312203.

[32] P. Bertet, S. Osnaghi, A. Rauschenbeutel, G. Nogues, A. Auffeves, M. Brune, J. M. Raimond and S. Haroche, Nature **411**, 166 (2001).

[33] A. Auffeves, P. Maioli, T. Meunier, S. Gleyzes, G. Nogues, M.Brune, J. M. Raimond and S. Haroche, Phys. Rev. Lett. **91**, 230405 (2003).

[34] J. Gea-Banacloche, Phys. Rev. Lett. **65**, 3385 (1990).

[35] J. Gea-Banacloche, Phys. Rev. A **44**, 5913 (1991).

[36] S. J. D. Phoenix and P. L. Knight, Phys. Rev. Lett. **66**, 2833 (1991).

[37] S. J. D. Phoenix and P. L. Knight, Phys. Rev. A **44**, 6023 (1991).

[38] M. V. Berry, "Chaos and the semiclassical limit of Quantum mechanics (is the moon there when somebody looks?)", in *Quantum Mechanics: Scientific perspectives on divine action* (Robert John Russell, Philip Clayton, Kirk Wegter-McNelly and John Polkinghorne Eds.), (Vatican Observatory CTNS publications, Berkeley, CA, 2001), pp. 41-54; Physics Today, May 2002, pp. 10-11.

[39] D. Bohm, *Quantum Theory*, (Dover, New York, 1951). The treatment of the Stern-Gerlach experiment through a singular limit is given in pp. 593 and following.

[40] H. M. Pastawski, P. R. Levstein, and G. Usaj, in *Some Contemporary Problems of Condensed Matter Physics In Contemporary Fundamental Physics*, Valerie V. Dvoeglazov Ed., (Nova, New York, 2001).

[41] H. M. Pastawski, P. R. Levstein, G. Usaj, J. Raya, and J. Hirschinger, Physica A **283**, 166 (2000).

[42] P. R. Levstein, G. Usaj, and H. M. Pastawski, Molecular Physics **95**, 1229 (1998).

[43] P. R. Levstein, G. Usaj, and H. M. Pastawski, J. Chem. Phys. **108**, 2718 (1998).

[44] P. R. Levstein, G. Usaj, and H. M. Pastawski, cond-mat/9803047.

[45] L. P. Kadanoff, *Statistical Physics*, (World Scientific, Singapore, 2000).

[46] M. Frasca, Phys. Rev. A **45**, 43 (1992).

[47] M. Frasca, Phys. Rev. A **47**, 2374 (1993).

[48] M. Frasca, Phys. Rev. A **58**, 3439 (1998).

[49] M. Frasca, Phys. Rev. A **60**, 573 (1999).

[50] A. V. Oppenheim and R. W. Schafer, *Discrete-Time Signal Processing*, (Prentice Hall, Englewood Cliffs, NJ, 1999).

[51] C. Rovelli, "Loop Quantum Gravity", Living Rev. Relativity **1**, 1, (1998). This can be found at http://www.livingreviews.org/lrr-1998-1.

[52] R. H. Dicke, Phys. Rev. **93**, 99 (1954).

[53] K. Hepp and E. Lieb, Ann. Phys. **76**, 360 (1973).

[54] K. Hepp and E. Lieb, Phys. Rev. A **8**, 2517 (1973).

[55] C. Emary and T. Brandes, Phys. Rev. Lett. **90**, 044101 (2003).

[56] C. Emary and T. Brandes, Phys. Rev. E **67**, 066203 (2003).

[57] T. Holstein and H. Primakoff, Phys. Rev. **58**, 1098 (1940).

[58] W. P. Schleich, *Quantum Optics in Phase Space*, (Wiley-VCH, Berlin, 2001).

[59] B. L. Altshuler, A.G. Aronov, and P. A. Lee, Phys. Rev. Lett. **44**, 1288 (1980); B. L. Altshuler and A. G. Aronov, in *Electron-Electron Interactions in Disordered Systems*, edited by A. L. Efros and M. Pollak (North-Holland, Amsterdam, 1985).

[60] J. J. Lin and N. Giordano, Phys. Rev. B **35**, 545 (1987).

[61] J. J. Lin and N. Giordano, Phys. Rev. B **35**, 1071 (1987).

[62] T. Hiramoto, K. Hirakawa, Y. Iye and T. Ikoma, Appl. Phys. Lett. **54**, 2103 (1989).

[63] E. Lieb and D. Mattis, Phys. Rev. **125**, 164 (1962).

[64] B. Sampedro, P. Crespo, A. Hernando, R. Litrán, J. C. Sánchez López, C. López Cartes, A. Fernandez, J. Ramírez, J. González Calbet, and M. Vallet, Phys. Rev. Lett. **91**, 237203 (2003).

[65] F. T. Arecchi, E. Courtens, R. Gilmore, and H. Thomas, Phys. Rev. A **6**, 2211 (1972).

[66] J. J. Lin and L. Y. Kao, J. Phys.: Cond. Matt. **13**, L119 (2001).

[67] N. Shannon and A. V. Chubukov, Phys. Rev. B **65**, 104418 (2002).

[68] K. Huang, *Statistical Mechanics*, (Wiley, New York, 1963).

[69] C. L. Siegel and J. K. Moser, *Lectures on Celestial Mechanics*, (Springer, Berlin, 1971).

[70] L. Onsager, Phys. Rev. **65**, 117 (1944).

[71] T. D. Lee and C. N. Yang, Phys. Rev. **87**, 404 (1952).